# Recipe Makes Sense

The popular steps in a recipe
- Arrange $k$ of $n$ objects
- Choose $k$ of $n$ objects    ($n$ combination)

**In how many ways can we <u>arrange</u> $k$ of $n objects?</u>**

<u>Answers:</u>

$$\underbrace{n \times (n-1) \times (n-2) \ldots \times (n-k+1)}_{k \text{ terms}} = \frac{n!}{(n-k)!}$$

(terms labeled 1, 2, 3, ..., $k$)

**In how many ways can we <u>choose</u> $k$ of $n$ objects?**

<u>Answer:</u>

Reads:
→ $n$ choose $k$.

Let $\binom{n}{k}$ be the number of ways to choose $k$ of $n$ objects.

$\binom{9}{1} = 9$

$\binom{9}{0} = 1$

$\binom{9}{9} = 1$

$\binom{9}{8} = \binom{9}{1} = 9$

$\binom{9}{5} = \binom{9}{4}$

**Formula.**

formula for $\binom{n}{k}$ by counting in how many ways we can arrange $k$ of $n$ objects.

$\rightarrow n(n-1)(n-2)\ldots(n-k+1) \leftarrow$ now count,

Step 1: Choose $k$ of $n$ objects. ← $\binom{n}{k}$ ways

Step 2: Arrange those chosen $k$ objects ← $k!$

The Answer is $\binom{n}{k} k! =$

Thus, $\binom{n}{k} k! = n(n-1)(n-2)\ldots(n-k+1)$, so $\binom{n}{k} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k!}$

$$= \frac{n!}{k!(n-k)!}$$

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$$\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = \frac{n(n-1)(n-k+1)}{k!} = 84$$

## More Examples:

How many positive 4 digit integers can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are there?

Answer: $9^4 = 9 \times 9 \times 9 \times 9$

(b) how many integers in part A has at least on 1's.

Answer: $\underbrace{4 \cdot 9 \cdot 9 \cdot 9}_{wrong}$ $\longrightarrow$ does not work for 1213 or any number with two of the same digits.